

# Exploration of Characteristic Equation Towards the Analysis of Dynamical Stability for Synchronous Generators through Swing Equation

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**Abstract** – This study reports a didactic approach on how to exploit the knowledge of the characteristic equations of any second-order system towards their stability analysis. To prove the efficacy of the method in industrial applications, the manuscript focuses on the stability study of synchronous generators. To obtain the future-ready characteristic equation, the dynamic equation of a synchronous generator is modeled via the swing equation. Literally, the swing equation describes the behavior of the rotor dynamics in the generator. By using the characteristic equation at hand, dominant parameters that affect the stability are identified. The analysis studies are then conducted to observe the stability through multifarious allowable parameters' range that disturb the roots (or the poles) location of the equation. The outcome of this research shows that two dominant parameters, named synchronizing coefficient and per unit inertia constant, determine the damping ratio and natural frequency. As a result, these two parameters affect the stability and transient characteristics of synchronous generators.

**Keywords:** inertia constant, rotor angle, swing equation, synchronizing coefficient, synchronous generator

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## I. Introduction

Any physical system that is linear and time-invariant can be modelled in either the time domain or the frequency domain. The dynamics of such systems are bounded in a class of linear time-invariant (LTI) systems if they adhere to the properties of linearity and time-invariant [1]. For a single-input-single-output system, a transfer function is appropriate to represent its dynamic. As such, knowing the ratio of the output to the input is just sufficient to represent its dynamic in the frequency domain, namely a transfer function [2], [3]. For a system having an input  $r(t)$  and the output  $c(t)$ , the transfer function defines the ratio  $\mathcal{L}[c(t)]/\mathcal{L}[r(t)]$ . Setting the denominator of the expression  $\mathcal{L}[c(t)]/\mathcal{L}[r(t)]$  to zero yields the characteristic equation of the system. If the system at hand is represented in the state variable  $\dot{x} = Ax$ , where the state  $x \in \mathbb{R}^n$  and the matrix  $A \in \mathbb{R}^{n \times n}$  be the  $n$ -order system, the transfer function can be easily formulated as  $G(s) = C(SI - A)^{-1}B$  when the system output is defined as  $y = Cx$ ,  $\forall C \in \mathbb{R}^{1 \times n}$ . The knowledge of characteristic

equations is useful to observe the stability and transient performance of the system. For a second-order system, the characteristic equation provides root behavior that reflects the damping factor of natural frequency. These criteria hence describe the transient characteristics that facilitate the control system or stabilizer design.

This manuscript reports the didactic method for analyzing the stability and transient criteria of a synchronous generator through the observation of characteristic equations. Beforehand, the synchronous generator must be modelled to obtain its dynamic equation. In a synchronous generator, the relative motion between the rotor axis and the synchronously rotating stator field axis is helpful in analyzing stability. Any changes in rotor angle result in changes in real power, which eventually affect the operating frequency. As a dynamic model is essential to studying the rotor angle stability of a synchronous generator, the swing equation is exploited to reach the main research objectives. The swing equation serves as the basis for modelling as it represents the equilibrium condition between mechanical power

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from the prime mover and electrical power for the end user.

There are many methods to obtain a mathematical model of synchronous generators. One of the established methods is the Park's transformation approach [4], [5]. The transformation is carried out to analyze the transient stability of 9 buses and 3 machines system. The author in [4] presents a detailed models of a synchronous generator that consists of a machine model, excitation, and prime mover controllers in per-unit. The methodology demonstrates the augmentation of an automatic generation control to guarantee the stability of power systems in the appearance of generator rotor swings. More treatment on automatic generation control can be found in [6], [7]. Instead of applying highly mathematical modeling, the parameters of synchronous generators can be estimated by using the least squares method [8], and the Taylor expansions approach [9]. However, the shortcoming of estimating the parameters incurs an insufficient knowledge of the transient characteristics of such generators. More recent research that utilizes the swing equation in the analysis of transient stability of synchronous generators can be found in [6], [10], [19], [20], [11]–[18]. In [11], the literature therein utilizes the equal area criterion method to an approximate one-machine infinite bus system. The approach seems unrelaxed the high mathematical computation in the modelling. Whereas in [21], the swing equation is modelled for the purposes of transient analysis and power deficit estimation, respectively.

The rest of the manuscript discusses the modeling concept of the swing equation, the analysis of stability and transient characteristics via the characteristic equation of the swing equation, and the analysis of research findings. Lastly, the outcome of the research is then concluded.

## II. Development of Swing Equation

The swing equation describes the behavior of the rotor dynamics in the generator. It is an equation describing the relative rotation of the rotor axis with the axis of time filed synchronously [22]. Whereas in [23], the author emphasized that this equation provides the relative motion, or acceleration. Both indicate the same meaning, where this equation shows the electromechanical oscillations or dynamics in a power system. In this phase, the swing equation is formulated based on the law of rotation in a synchronous generator, as depicted in Fig. 1 [24].

From Newton's Second Law of Rotation,  $T = J\alpha$ , the swing equation governs the motion of the rotor by the combined moment inertia,  $J$  of prime mover and generator with respect to rotor acceleration,  $\alpha$ . The dynamic of the synchronous generator can be presented as follows [25]:

$$J \frac{d^2 \delta_m}{dt^2} = T_m - T_e \quad (1)$$

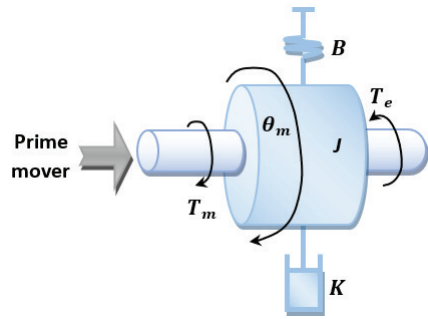


Fig. 1. Synchronous generator – Concept of torque balance

Rotor acceleration,  $\alpha$ , is the second derivative of rotor angular displacement,  $\theta_m = \omega_{sm}t + \delta_m$ , with respect to the stationary reference axis on the stator, as shown in Fig. 2.

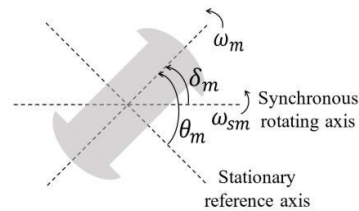


Fig. 2. Rotor angular displacement

Multiplying torque by rotor speed,  $\omega_m$ , produces power; as such, the swing equation in (1) can be represented in terms of power as in (2).  $J\omega_m$  is known as inertia constant denoted as  $M$ , thus the swing equation is written in terms of inertia constant as in (3).

$$J\omega_m \frac{d^2 \delta_m}{dt^2} = P_m - P_e \quad (1)$$

$$M \frac{d^2 \delta_m}{dt^2} = P_m - P_e \quad (2)$$

The inertia constant,  $M$  is related to the kinetic energy of rotating masses,  $W_k$ , by  $M = 2W_k/\omega_m$ .  $M$  is not really constant since the rotor speed may vary from synchronous speed. Due to the fact that rotor speed,  $\omega_m$  does not differ significantly from synchronous speed,  $\omega_{sm}$  when the generator is stable,  $M$  is considered to remain constant and evaluated at synchronous speed as  $M = 2W_k/\omega_{sm}$ . The swing equation in (3) is more convenient to be written in terms of electrical power angle,  $\delta$  as:

$$\frac{2}{p} M \frac{d^2 \delta}{dt^2} = P_m - P_e \quad (3)$$

in which  $p$  is number of poles and electrical power angle,  $\delta$ , is correlated to mechanical power angle,  $\delta_m$ , by  $\delta = \frac{p}{2} \delta_m \Rightarrow \delta_m = \frac{2\delta}{p}$ . Conventionally, power system analysis is conducted in per unit system, thus, the swing equation is expressed as:

$$\frac{2}{p} \left( \frac{2W_k}{\omega_{sm} S_B} \right) \frac{d^2 \delta}{dt^2} = \frac{P_m}{S_B} - \frac{P_e}{S_B} \quad (4)$$

In stability studies, another important constant is the per unit inertia constant,  $H$ , which is defined as the ratio of kinetic energy stored at rated speed in  $MJ$  to the generator rating in  $MVA$  or expressed by  $H = W_k / S_B$ . The value of  $H$  is in the range of 1 to 10 seconds, depending on the size and type of the machine. Substituting  $H$  into (5) renders:

$$\frac{2}{p} \frac{2H}{\omega_{sm}} \frac{d^2 \delta}{dt^2} = P_m(pu) - P_e(pu) \quad (5)$$

where  $P_m(pu)$  and  $P_e(pu)$  are the per unit mechanical power and per unit electrical power, respectively. The  $pu$  in (6) is omitted to simplify the notation. Electrical velocity and mechanical angular velocity are related by  $\omega_{sm} = (2/p)\omega_s$ . Thus, (6) can be expressed in terms of electrical angular velocity, frequency, and electrical degree as in (7), (8) and (9), respectively.

$$\frac{2H}{\omega_s} \frac{d^2 \delta}{dt^2} = P_m - P_e \quad (6)$$

$$\frac{H}{\pi f_0} \frac{d^2 \delta}{dt^2} = P_m - P_e, \delta \text{ in electrical radian} \quad (7)$$

$$\frac{H}{180 f_0} \frac{d^2 \delta}{dt^2} = P_m - P_e, \delta \text{ in electrical degrees} \quad (8)$$

### III. Analysis of Stability

In order to conduct stability analysis, a single machine connected to an infinite bus is considered. The single machine is a three-phase, non-salient, two-pole rotor of a synchronous generator. According to [26] his model is very representative of power system and is widely used in power system distribution simulation. The machine's output electrical power is expressed as:

$$P_e = \frac{|E'| |V|}{X_{12}} \sin \delta \quad (9)$$

Equation (10) shows that the electrical power produced depends on the transfer reactance,  $X_{12}$ , and the angle between the two voltages, which also the called rotor angle,  $\delta$ . The relation can be expressed by a power angle curve, as shown in Fig. 3.

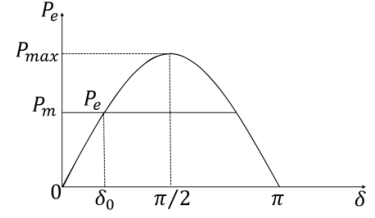


Fig. 3. Power angle curve

The curve shows that the generator power output can be gradually increased until maximum power,  $P_{max}$ , is transferred. According to [25], the maximum power,  $P_{max}$ , which is referred to as the steady-state stability limit, occurs at an angular displacement of  $90^\circ$  is expressed as  $P_{max} = \frac{|E'| |V|}{X_{12}}$ . Electrical power output,  $P_e$  will decrease from  $P_{max}$  point if the rotor angle,  $\delta$ , is advanced further by further increasing the shaft input. The electrical power equation in terms of  $P_{max}$  is  $P_e = P_{max} \sin \delta$ .

The electrical power equation is substituted into (8), renders (11) such that the analysis of stability can be performed.

$$\frac{H}{\pi f_0} \frac{d^2 \delta}{dt^2} = P_m - P_{max} \sin \delta \quad (10)$$

The swing equation is a nonlinear function of the power angle. Consider the deviation,  $\Delta\delta$ , in power angle from initial operating point,  $\delta_0$ , that is  $\delta = \delta_0 + \Delta\delta$ , the swing equation becomes

$$\frac{H}{\pi f_0} \frac{d^2 \delta_0 + \Delta\delta}{dt^2} = P_m - P_{max} \sin(\delta_0 + \Delta\delta)$$

$$\frac{H}{\pi f_0} \frac{d^2 \delta_0}{dt^2} + \frac{H}{\pi f_0} \frac{d^2 \Delta\delta}{dt^2} = P_m - P_{max} (\sin \delta_0 \cos \Delta\delta + \cos \delta_0 \sin \Delta\delta) \quad (11)$$

In obtaining the characteristic equation of the swing equation, nonlinear swing equation (12) is linearized by assuming the rotor deviation,  $\Delta\delta$ , is very small, which leads to  $\cos \Delta\delta \approx 1$  and  $\sin \Delta\delta \approx \Delta\delta$ . Hence, (12) is simplified as:

$$\frac{H}{\pi f_0} \frac{d^2 \delta_0}{dt^2} + \frac{H}{\pi f_0} \frac{d^2 \Delta\delta}{dt^2} = P_m - P_{max} \sin \delta_0 - P_{max} \cos \delta_0 \Delta\delta \quad (12)$$

By comparing (13) to (11), the linearized equation in incremental changes in power angle is described as:

$$\frac{H}{\pi f_0} \frac{d^2 \Delta\delta}{dt^2} = -P_{max} \cos \delta_0 \Delta\delta \quad (13)$$

Note that  $P_{max} \cos \delta_0$  is the slope of the curve in the power angle graph at  $\delta_0$ , which also known as the synchronizing coefficient, denotes by  $P_s$ , (14) is rewritten as:

$$\frac{H}{\pi f_0} \frac{d^2 \Delta \delta}{dt^2} + P_s \Delta \delta = 0 \quad (14)$$

Taking Laplace transform of (15),

$$\begin{aligned} \mathcal{L} \left\{ \frac{H}{\pi f_0} \frac{d^2 \Delta \delta}{dt^2} \right\} + \mathcal{L} \{ P_s \Delta \delta \} &= 0 \\ \frac{H}{\pi f_0} s^2 \Delta \delta(s) + P_s \Delta \delta(s) &= 0 \\ \left[ \frac{H}{\pi f_0} s^2 + P_s \right] \Delta \delta(s) &= 0 \end{aligned}$$

Hence, the root locations of the linearized equation are as in (16).

$$s^2 = -\frac{\pi f_0}{H} P_s \quad (15)$$

Further analysis conducted by adding damping power to (16) renders (17). The damping power is an important function that minimizes the difference between the two angular velocities, thus damping out the oscillation of the transient response [27], [28]. Damping power is approximately proportional to the speed deviation.

$$\frac{H}{\pi f_0} \frac{d^2 \Delta \delta}{dt^2} + D \frac{d \Delta \delta}{dt} + P_s \Delta \delta = 0 \quad (16)$$

In terms of standards characteristic equation,  $\frac{d^2 \Delta \delta}{dt^2} + 2\zeta \omega_n \frac{d \Delta \delta}{dt} + \omega_n^2 \Delta \delta = 0$ , (17) can be expressed as:

$$\frac{d^2 \Delta \delta}{dt^2} + \frac{\pi f_0}{H} D \frac{d \Delta \delta}{dt} + \frac{\pi f_0}{H} P_s \Delta \delta = 0 \quad (17)$$

Thus, natural frequency and damping ratio are derived as:

$$\text{Natural frequency, } \omega_n = \sqrt{\frac{\pi f_0}{H} P_s} \quad (18)$$

$$\text{Damping ratio, } \zeta = \frac{D}{2} \sqrt{\frac{\pi f_0}{H P_s}} < 1 \quad (19)$$

The damping ratio is required to be less than 1 or underdamped; as such, the response will oscillate through equilibrium and ensure the system reaches the desired end state with some overshoot. It is observed that there exist two dominant parameters, known as the synchronizing coefficient,  $P_s$ , and the inertia constant,  $H$ , that determine the pole's location, natural frequency and damping ratio.

#### IV. Simulation Result and Discussion

This section validates the dominant parameters effect on stability and transient characteristics via simulation. The simulation is carried out in a MATLAB/Simulink environment.

The test is conducted for the synchronizing coefficient,  $P_s$ , with values of 1 and -1. When the value of  $P_s$  is -1 (negative), there exists only one pole on the right half of the s-plane, which causes the response to be exponentially increased and hence lose stability, as shown in graph of rotor angle deviation and frequency response in Fig. 4(a) and (b), respectively. When the value of  $P_s$  is 1 (positive), there exist two poles on the  $j\omega$ -axis with oscillatory and undamped motion. Thus, the system is said to be marginally stable. The transient response of rotor angle deviation and frequency is as shown in Fig. 5(a) and (b), respectively. This result shows that  $P_s$  affects the root location as well as the stability of the system.

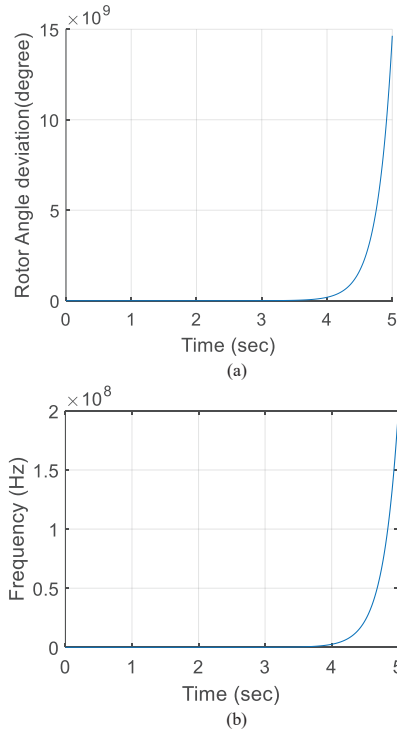


Fig. 4. Transient response when  $P_s = -1$

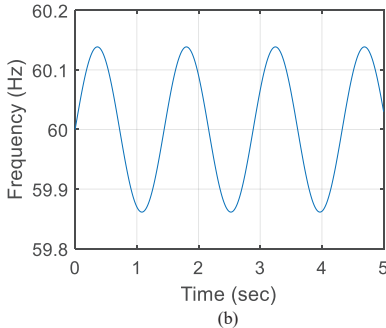
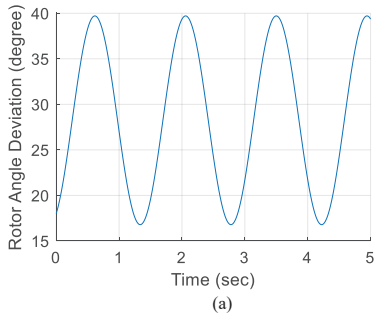


Fig. 5. Transient response when  $P_s = 1$

The following test is conducted with damping power added to the linearized swing equation. Fig. 6 and Fig. 7 show the transient responses of rotor angle deviation and frequency, respectively, when the value of  $H$  is varied ( $H = 3, 5, 7, 9$ ). When  $H$  increased, the natural frequency and damping ratio are decreased, which results in a longer settling time. The increment of  $H$  results in a transient response that becomes less steep and more damped. This proves that  $H$  is significantly affects the transient response.

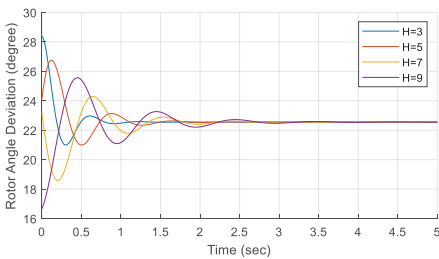


Fig. 6. Transient response of rotor angle deviation when  $H = 3, 5, 7, 9$

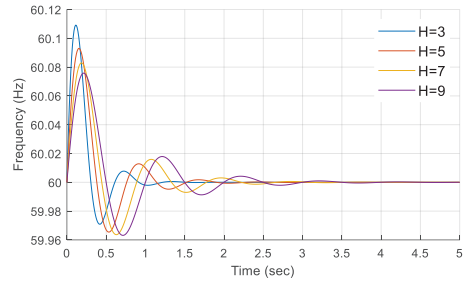


Fig. 7. Transient response of frequency when  $H = 3, 5, 7, 9$

## V. Conclusion

This study reports the stability analysis of the characteristic equation for second-order system of a synchronous generator. The simulation result shows the synchronizing coefficient,  $P_s$  affects the root location as well as the stability of the system. Furthermore, adding damping power to the swing equation results in underdamped oscillation and improved stability. The increment of the inertia constant,  $H$  shows a varies transient response with less steep and more damp. As a conclusion, the synchronizing coefficient,  $P_s$  and inertia constant,  $H$  affect the transient characteristic and stability of the synchronous generator.

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## Conflict of Interest

The authors declare no conflict of interest in the publication process of the research article.

## Author Contributions

N. S. F. M. Murad designed the experimental cases, conducted the analysis, and wrote the original draft. M. N. Kamarudin conceptualized the research, provided full supervision throughout the research, continually reviewed the process until finalizing results, and edited the paper. A. N. Hanafi and S. M. Rozali reviewed the draft and edited the paper. M. A. Ibrahim conducted proofreading and grammar checking.

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