# Synchronise Control of Chua's Chaotic Circuits

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**Abstract** – *Chua circuit is a nonlinear circuit that shows complex dynamical behaviour including chaos. In chaos theory, the principle is known as 'Butterfly Effect' where a small change in the initial condition will cause large difference in the result. In this paper, the range of value of a resistor that cause chaotic behaviour is identified. A mathematical modelling is derived from the Chua's circuit and represented in state-space equation. Then, a state-feedback controller is conFig.ured to stabilise and synchronise the slave system for two cases of reference input, that are, constant and chaotic signals. The output results showed the slave system follows the master system's behaviour with zero synchronisation error.*

***Keywords***: *Chaos, Chua circuit, Synchronisation, Nonlinear system*

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## I. Introduction

What is chaos exactly? When engineers use the word chaos, they will imagine unpredictable results. It is tough to define chaos but it is easy to "recognize it" [1]. The easiest way to observe chaos is in Chua's circuit. History about chaos was started in 1889 when King Oscar II of Sweden had a question about 'three bodies problem' of planets in a contest. Then, a mathematician named Henri Poincare won the prize, which he discovered that orbit of three or more interacting celestial bodies that exhibits unpredictable behaviour. Thus, chaos is born [2].

In 1963, Edward Lorenz published a journal with title "Deterministic Nonperiodic Flow" [3] and he was credited for the Chaos Theory. In [4], a calculation about three decimal places which yield a different outcome of the model. When the process repeated many times, the authors found out that there is a difference result each time. The principle of Sensitive Dependence on Initial Conditions (SDIC) is discovered where it is the key component in a chaotic system. This yields a "Butterfly Effect" that is known as Lorenz attractor as shown in Fig. 1.

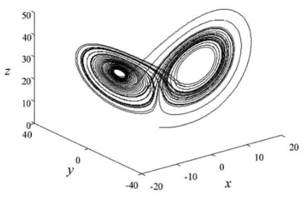



Fig. 1. Butterfly Effect [5].

In 1980, Ueda and Akamatsu found a chaos in negative resistance oscillator namely Chua circuit. This circuit was widely used for experiment in controlling chaotic system, see for example works in [5]-[7]. Researcher and scientist starting to design a circuit model as shown in Fig. 2 that contains an inductor, two capacitors, a linear resistor, and a nonlinear resistor [8]. By applying the Kirchhoff's law, Chua's circuit is described by three differential equations:

$$C_1\frac{dv_{c1}}{dt} = G(V_{C2} - V_{C1}) - g(V_{C1}) \tag{1}$$

$$C_1\frac{dv_{c2}}{dt} = G\ (V_{C1} - V_{C2}) + I_L \tag{2}$$

$$L\frac{di_L}{dt} = -V_{C2} \tag{3}$$

where

$$g(v_{c1}) = m_o v_R + \tfrac{1}{2}\ (m_1 - m_0)\ [|v_R + B_P| - |v_R - B_P|] \tag{4}$$

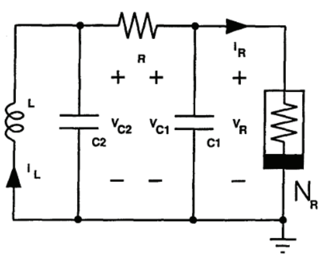



Fig. 2. Chua Circuit [9].

where $m_0$ and $m_1$ are the slopes in the inner and outer regions, and $\pm B_P$ denote the breakpoints. The nonlinear resistor generates a piecewise function represented in Fig. 3.

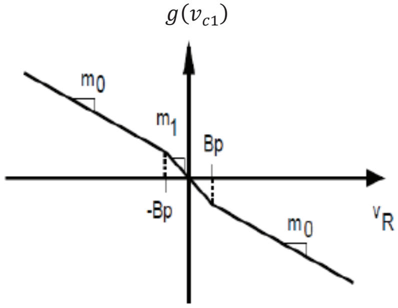



Fig. 3. Chua's nonlinear resistor function [9].

The objectives of the present paper are to identify the range of resistance that exhibits chaos in the system's dynamic and to validate the state-space model in the MATLAB-simulation environment. This paper proposes a simple state feedback control via pole-placement to stabilize the system trajectories and synchronize two systems for a certain initial conditions and set-points.

## II. Literature Review

Chaos system is a dynamical system that highly sensitive to initial condition and discovered by Edward Norton Lorenz in weathers model. In this system, he used Lyapunov exponent method to measure the divergence of nearby trajectories and it can quantify the chaotic systems. A positive Lyapunov exponent and phase space compactness will be determined to observe the chaotic system. Chaos behaviour can be found in many electrical systems, for example in electronic circuits and telecommunication systems.

An electronic circuit consists of passive network and nonlinear elements, such as infinite impulse response (IIR) digital filter and oscillator circuit display chaos and bifurcation behaviours. The common goal of control for chaotic electronic circuits is suppression of oscillations and production of stable periodic or constant motion. Two methods of feedback controller known to stabilize the chaotic behavior are OGY and Pyragas methods. The former method uses feedback controller technique to eliminate chaos by shading one of the infinitely unstable periodic orbits in the chaotic attractor [9], while the latter has been developed to stabilize the periodic orbit by applying small time continuous into a parameter. It is known as delayed feedback control and evolve in continuous time [10].

Another goal could be synchronisation of the system to a different chaotic signal. Three forms of chaos synchronisation are chaotic masking, chaotic modulation, and chaotic switching. These applications are important in secure communication systems. In chaotic masking, the information signal is connected at the transmitter while at receiver, the original chaotic signal is reconstructed by using chaos synchronisation [11]. Meanwhile in chaotic modulation, the communication is based on modulated transmitter parameter which is information signal is combine with chaotic system for modulation. At receiver, the dynamics of chaotic signal is tracked for retrieve the information signal [12]. In chaotic switching, the working principle is to map bits or symbol to basic functions of chaotic signal to show from one or more chaotic attractors [13].

On the other hand, chaos synchronization for two systems known as master-slave system are classified as complete synchronisation, generalized synchronisation, projective synchronisation, phase synchronisation, lag synchronisation, impulsive synchronisation, and adaptive synchronisation. Details of these types of synchronisation can be referred in [14 – 21], respectively and the references therein. Besides that, there is also another method known as continuous control [22]. To synchronise two chaotic systems, system A and B are denoted as $x$ and $y$, respectively and described by

$$\frac{dx}{dt} = \dot{x} = f(x) \tag{5}$$

$$\frac{dy}{dt} = \dot{y} = f(y) \tag{6}$$

where $x, y \in R^n$. The configuration of Fig. 4 shows the systems are coupled in unidirectional form. $D(t)$ in the system is defined to compare the signal $x_i(t)$ and $y_i(t)$, where $i = 0, 1, 2, \ldots$ and the parameter is $K > 0$.

$$K[x_i(t) - y_i(t)] = KD(t) \tag{7}$$

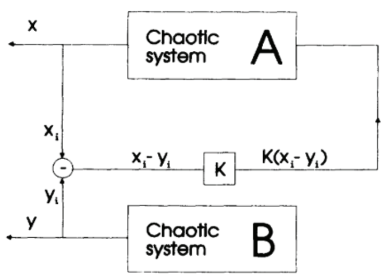



Fig. 4. Continuous control of synchronisation.

The advantages of this method are it can synchronise multidimensional system by feeding back a single variable and very convenient to apply. But the limitation is both number of positive Lyapunov exponents in coupled system and single systems must equal for synchronising.

## III. Methodology

This section explains the design of chaotic circuits including selection of resistance value, feedback control and synchronise control of the chaotic systems. The experiments started identifying the range of resistance of the Chua's circuit that yield a chaotic behaviour and then a resistance value is chosen to derive a mathematical model using the Kirchhoff's laws. The mathematical model is converted into dimensionless state space. The behaviour of attractor is observed, and the synchronising controller is designed in MATLAB-Simulink environment. A state-feedback controller is designed in via pole placement method to stabilise the system. Furthermore, a reference tracking for the feedback controller is designed for the system to converge and maintain at a set-point. The analysis is conducted by observing the system trajectories in terms of the steady state and synchronisation errors. Fig. 5 shows the flowchart of the experiments.

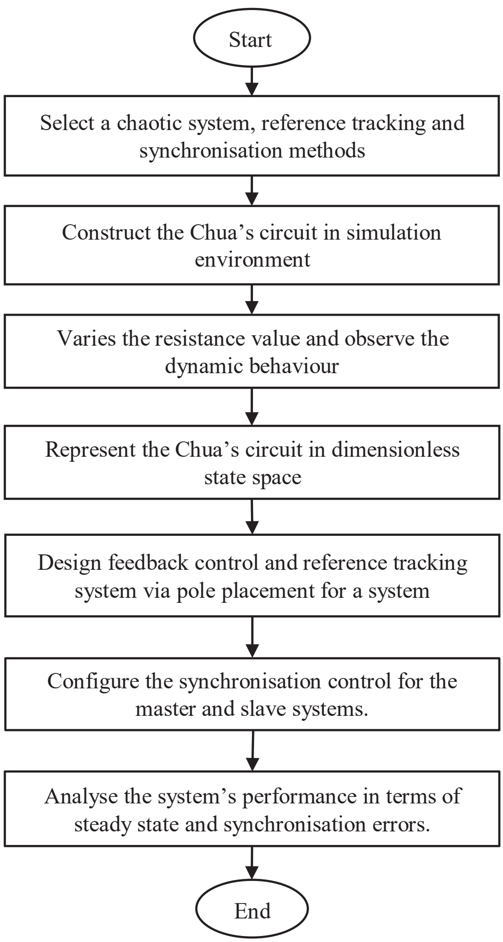



Fig. 5. Project flowchart.

*A. Chua's circuit modelling for master and slave systems*

Chua's circuit is modelled based in equation (1)-(4) to determine the relationship between the current and voltages of the circuit. It is observed that chaotic behaviour also depends on resistance value. Thus, the value of resistance, R is selected at 1.1kΩ, 1.4kΩ, 1.7kΩ, 2.0kΩ and 2.3kΩ. This is to identify at which value of resistance a chaotic attractor is produced.

The equations (1) – (4) in terms of passive elements are given as in equation (8)

$$\begin{bmatrix} -\frac{1}{RC_1} & \frac{1}{RC_1} & 0 \\ -\frac{1}{RC_2} & -\frac{1}{RC_2} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & 0 \end{bmatrix} \tag{8}$$

The Chua's nonlinear resistor function in Fig.. 3 is

$$f(Hx) = m_0 V_1 + \frac{1}{2}(m_1 - m_0)\left[|V_1 + B_p| - |V_1 - B_p|\right] \tag{9}$$

where $f(Hx)$ or $g(v_{c1})$ is the current through Chua's diode ($I_{NR}$). The voltage-controlled driving point characteristic is modelled as

$$I_{N_R} = \begin{cases} m_1 V_{c1} + (m_1 - m_0)B_p & V_{C1} < -B_p \\ m_0 V_{c1} & B_p \le V_{C1} \le -B_p \\ m_1 V_{c1} + (m_0 - m_1)B_p & V_{C1} > -B_p \end{cases} \tag{10}$$

The Chua's circuit parameters values are presented in Table I.

TABLE I
CHUA'S CIRCUIT PARAMETERS

| Parameter | Value |
|---|---|
| Inductor, $\boldsymbol{L}$ | 18 $mH$ |
| Capacitor, $\boldsymbol{C_1}$ | 10 $nF$ |
| Capacitor, $\boldsymbol{C_2}$ | 100 $nF$ |
| Breakpoint, $\boldsymbol{B_p}$ | ±1.186 |
| Inner slopes, $\boldsymbol{m_0}$ | −8/7 |
| Outer slopes, $\boldsymbol{m_1}$ | −5/7 |

The parameters of master and slave systems are the same except for the initial condition. A different initial condition is used to produce different pattern of chaotic signals between both systems.

On the other hand, for the Simulink application, equations (1) - (4) are converted into dimensionless form as shown in equations (11) - (14) to observe chaotic in time series analysis. Thus, it will be easier and more convenient if the set of differential equation is simplified into a dimensionless parameter ($\alpha, \beta$, a, b).

$$\frac{dx}{d\tau} = \alpha[-x + y - f(Hx)] \quad (11)$$

$$\frac{dy}{d\tau} = x - y + z \quad (12)$$

$$\frac{dz}{d\tau} = -\beta y \quad (13)$$

$$f(Hx) = a_0 x + \frac{1}{2}(a_1 - a_0)[|x+1| - |x-1|] \quad (14)$$

where

$$x = \frac{V_1}{B_p} \qquad y = \frac{V_2}{B_p} \qquad z = \frac{R_{i_L}}{B_p}$$

$$\alpha = \frac{C_2}{C_1} \qquad \beta = \frac{R^2 C_2}{L}$$

$$a_0 = |Rm_0| \qquad a_1 = |Rm_1|$$

Therefore, the state space equation for the master and slave systems has the form

$$\dot{x} = Ax + B \quad (15)$$

$$y = Cx \quad (16)$$

where

$$\text{A} = \begin{bmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & \beta & 0 \end{bmatrix} \quad \text{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{C} = [1 \quad 0 \quad 0]$$

In the next sections, two cases of synchronisation are presented. The first case considers a reference point tracking of the master-slave system, that is, the slave follows a constant output of the master system. The second case considers a chaos synchronisation, that is, the slave follows the chaotic signal of the master system.

### *B. Case 1: Set-point master-slave synchronisation*

A feedback controller is designed in the master and slave system to drive the systems to desired input state equilibrium points. The pole placement method is applied for the system to has close-loop poles at −15+j5, −15−j5 and −5. The initial conditions in master and slave systems are $[0.1 \quad 0.1 \quad 0.1]$ and $[0.3 \quad 0.3 \quad 0.3]$, respectively. The state equations are stated as in equations (17) – (20) where $m$ and $s$ denote master and slave systems, respectively.

$$\dot{x}_m = Ax_m + Bu_m \quad (17)$$

$$y_m = Cx_m \quad (18)$$

$$\dot{x}_s = Ax_s + Bu_s \quad (19)$$

$$y_s = Cx_s \quad (20)$$

where the system parameters are given as

$$A = \begin{bmatrix} -10 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -16 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C = [1 \quad 0 \quad 0].$$

Then, the parameters above are substituted into equation (21) to find the value of $K$.

$$\det(sI - (A - BK) = 0 \quad (21)$$

The reference set-point $x_{ref_m}$ of the master system is determined for the desired input value $u_{ref_m}$ of 0.5. The feedback controller of the system with reference tracking can written as shown below:

$$u_m = K\left(x_m - x_{ref_m}\right) + u_{ref_m} \quad (22)$$

$$u_s = K(x_s - x_m) + u_{ref_s} \quad (23)$$

where $K = [K_1 \quad K_2 \quad K_3]$.

Hence, the master and slave systems with reference tracking satisfy

$$\dot{x}_m = Ax_m + B\left[K\left(x_m - x_{ref}\right) + u_{ref_m}\right] \quad (24)$$

$$y_m = Cx_m \quad (25)$$

$$\dot{x}_s = Ax_s + B\left[K(x_s - x_m) + u_{ref_s}\right] \quad (26)$$

$$y_s = Cx_s \quad (27)$$

Fig. 6 illustrates the master-slave system with reference tracking state-feedback controller. The synchronisation error between the master and slave systems is observed at Scope z1 which the synchronization error is

$$e = x_s - x_m. \quad (28)$$

### *C. Case 2: Chaotic master-slave synchronisation*

In chaos synchronisation, the master system produces a chaotic signal which is fed to the slave system as the reference input. The slave system is to follow the behaviour of master system when the state-feedback controller is activated. The system configuration is illustrated in Fig. 7. The state-variable signals in both systems are compared in Scope 3 and the synchronisation error is observed at Scope 4. The error is fed back to the system via a controller.

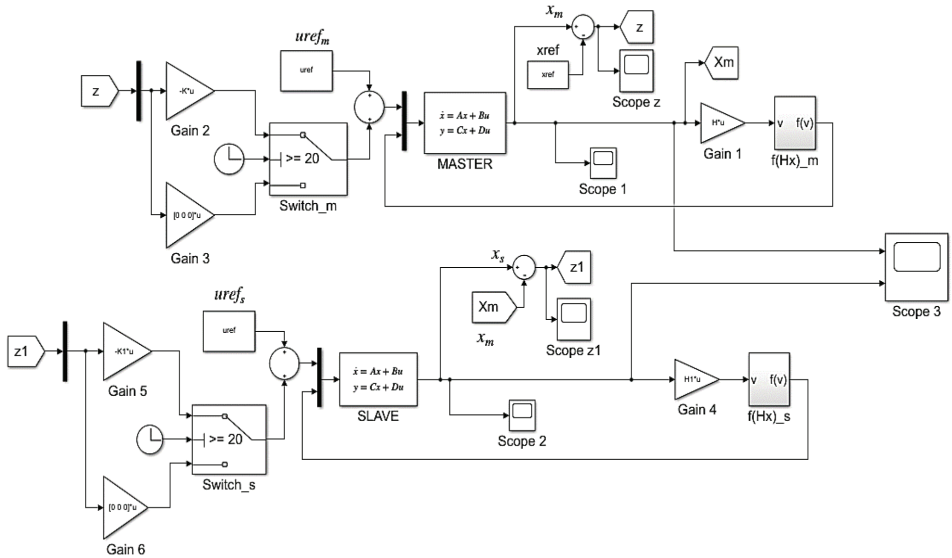



Fig. 6. Reference tracking in the master-slave system.

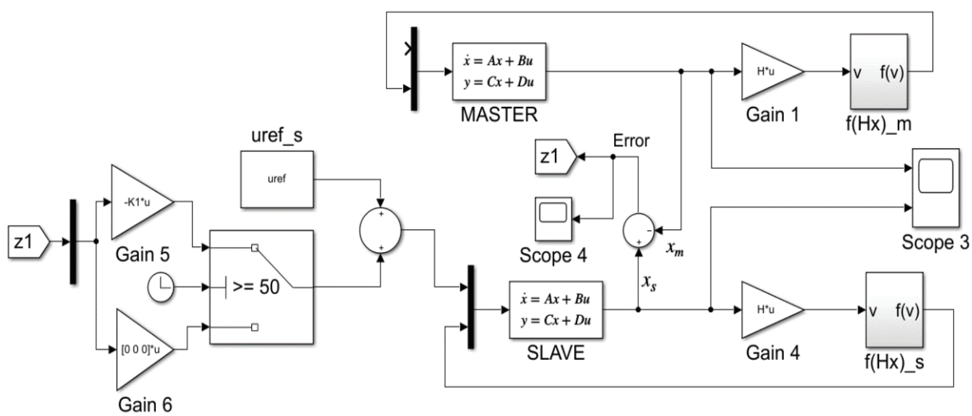



Fig. 7. Chaos synchronisation in the master-slave systems.

## IV. Result and Discussion

### A. Observation of chaotic behaviour

Chua's circuit is a non-linear circuit used to show chaotic behaviour in electrical system. The behaviour is induced because of capacitor, inductor, and a nonlinear resistor in the circuit. The resistor value is varied within a range $1.1 - 2.3\ k\Omega$ and the other parameters of Chua's circuit for master and slave system follow the value in Table 2 with the initial condition of $[-3 \quad -0.13 \quad 0.003]$. The types of behaviour observed is shown in Table II.

TABLE II
TYPES OF BEHAVIOUR

| Resistance value (kΩ) | Types of bifurcation | Types of behaviour |
|---|---|---|
| 1.1 | Limit cycle | Periodic |
| 1.4 | Limit cycle | Periodic |
| 1.7 | Double scroll attractor | Chaotic |
| 2.0 | Fixed point | Equilibrium |
| 2.3 | Fixed point | Equilibrium |

Fig. 8 shows that double scroll attractor exhibited for $R = 1.7k\Omega$.

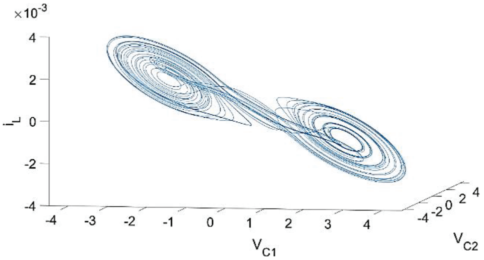


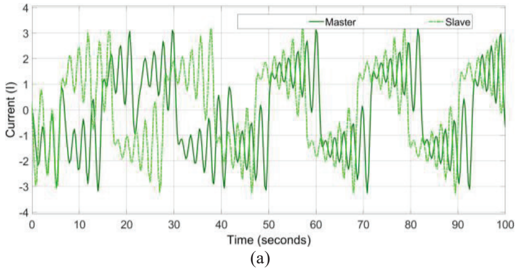

Fig. 8. Double scroll attractor at R=1.7 kΩ.

Double scroll attractor exhibits when two spirals points of attraction is curled up and form infinite rotations. The trajectory in the attractor starts to rotate at the upper one in anti-clockwise direction. When the rotation is continued, the trajectory move further until at one point it will not return to a starting point and descend downward to $V_{c1}$ axis in spiral paths on the lower part of the attractor. This causes the attractor to rotate again in anti-clockwise around the lower hole and produces lower attractor. Thus, the behaviour shows the same pattern as in the upper part of the attractor.

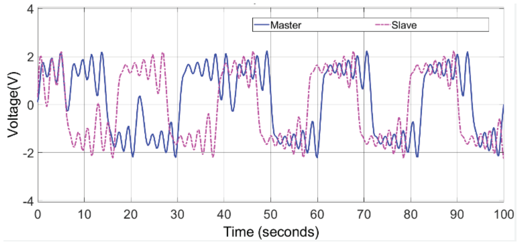
### *B. Chaotic behaviour in Simulink*

In Simulink, the chaotic behaviour is observed in time real-time. The initial condition of the master system is [ 0.1 0.1 0.1] and the slave system is [0.3 0.3 0.3].

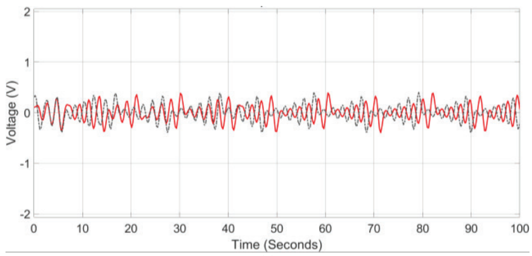
Fig. 9 shows a chaos signal in both systems for the given initial conditions. The signals between the master and slave systems are compared for the three state variables, that are the current in inductor, and voltages of capacitor 1 and capacitor 2. These signals exhibit a different pattern but periodic data for different initial condition.




(a)




(b)




(c)

Fig. 9. Chaos in master and slave for (a) inductor, (b) capacitor 1, (c) capacitor 2.

The voltage-current relationship of the Chua's diode is shown in Fig. 10. The graph was divided into three segments of piecewise-linear that are, two outers ($m_0$) and one inner ($m_1$) negative slope with two breakpoints ($\pm B_p$). The variables of the nonlinear representation in equation (14) are $m_0 = -8/7$, $m_1 = -5/7$, $\pm B_p = \pm 1.186$.

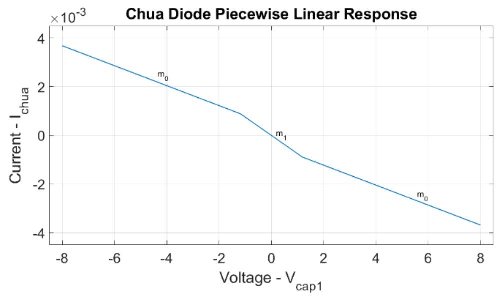
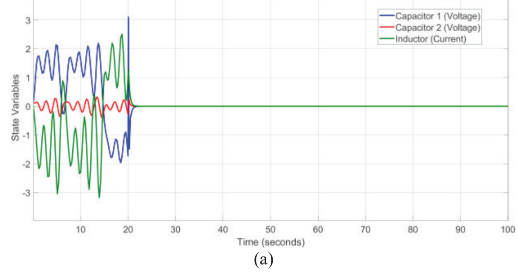


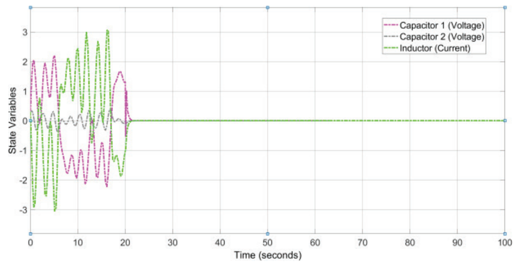

Fig. 10. V-I characteristics of Chua's diodes.

### *C. Case 1: Set-point master-slave synchronization*

Feedback controller via pole placement method was designed in master and slave circuits to stabilise the chaotic system. A feedback gain of $K = [24 \quad 36 \quad -44.12]$ is selected to place the closed-loop poles at $-15 + j5$, $-15 - j5$ and $-5$. Fig. 11 shows the signals of the master and slave systems started from a chaotic to zero steady state after controller activated at 20 seconds.




(a)

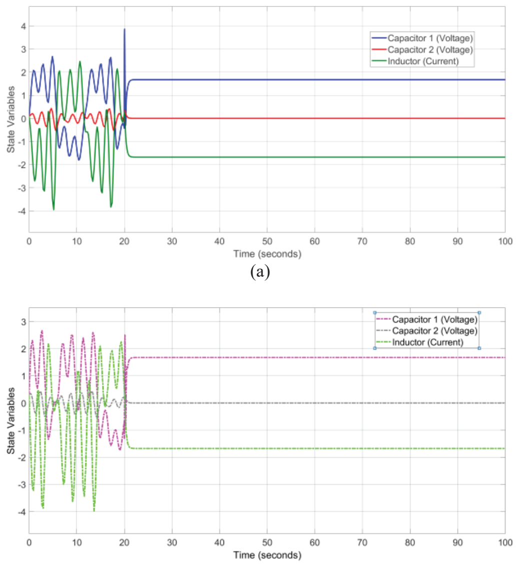



(b)

Fig. 11. Feedback gain in (a) master system (b) slave system.

A reference input $(x_{ref}, u_{ref})$ is applied to the systems for the feedback controller to track the desired reference and at the same time stabilize the system with desired requirement. The reference input is $x_{ref} = [1.6750 \quad 0 \quad -1.6750]$ with control input reference, $u_{ref} = 0.5$.

Fig. 12 showed that the state variables track when the reference value $x_{ref}$ the controller is activated at $t = 20$ seconds. Both output trajectories converge to the desired reference value.




(b)

Fig. 12. Reference tracking in (a) master system (b) slave system.

The synchronisation error between the master and slave systems is shown in Fig. 13. It can be observed that there is error before the controller is activated at $t = 20$ seconds and zero error after 20 seconds.

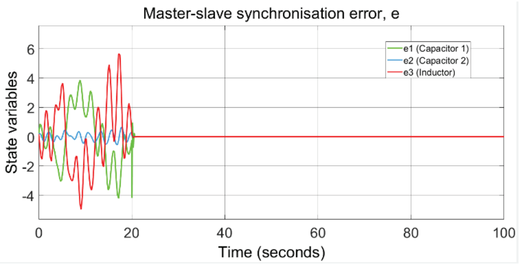
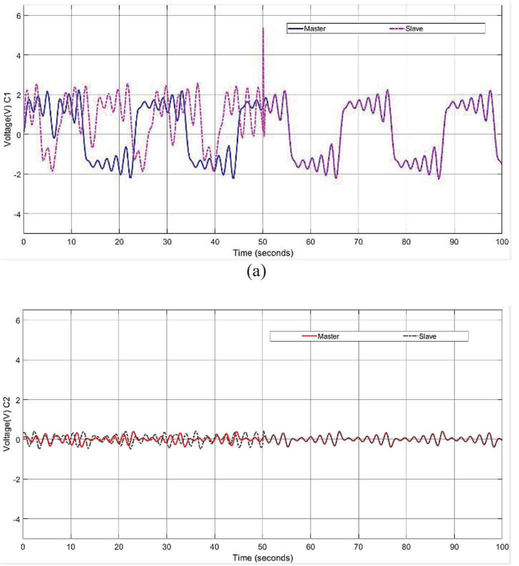



Fig. 13. Master-slave synchronisation error.

*D. Case 2: Chaotic master-slave synchronisation*

Chaotic synchronisation is defined as the trajectories of the slave system follows the master system's chaotic behaviour. Recall that, the behaviour of the master system is chaotic and unpredictable due to uncontrolled system. This output from the master system is used as reference input for the slave system. To synchronise the systems state feedback controller is applied in the slave system. A feedback gain of $K = [24 \quad 36 \quad -44.12]$ is selected to place the closed-loop poles at $-15 + j5$, $-15 - j5$ and $-5$.

Fig. 14 showed the chaos synchronisation between the master and slave systems at $t = 50$ seconds when the state feedback controller in the slave system is activated.

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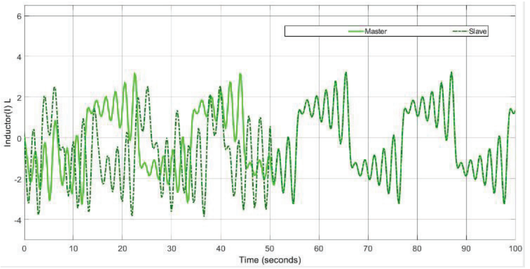


(c)

Fig. 14. Chaos synchronisation in (a) voltage of capacitor 1, (b) voltage of capacitor 2, and (c) current of inductor.

The synchronisation error can be observed by comparing the outputs of the master system $x_m$ and the slave system $x_s$. When the difference is zero, both systems is synchronised as shown in Fig. 15. The error signals before $t = 50$ seconds are between $\pm 6$ and it converges to zero error after the controller is activated.

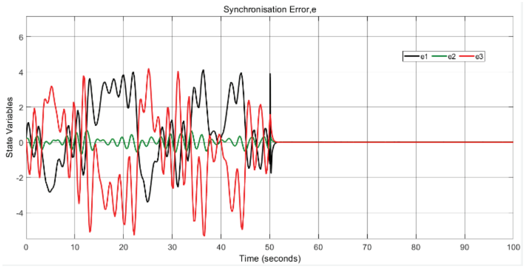


Fig. 15. Master-slave synchronisation error.

## V. Conclusion

In conclusion, Chua's circuit is modelled to determine the chaotic attractor and behaviour of the circuit. The parameters of chaotic attractor are chosen specifically to exhibit chaotic behaviour in phase portrait and time domain analysis. Thus, the type of attractor can be identified. In this paper, there are two identical Chua's circuits are used in designing synchronise chaotic control that are named as master and slave systems. The parameters for both systems are the same except the initial condition. This is to trigger different pattern of chaos for both circuits. Two cases of synchronisation control are considered, a) constant reference synchronisation, b) chaotic synchronisation. A state-feedback controller is implemented to achieve master-slave synchronisation. The results show the conFig.urations able to track the reference inputs with zero synchronisation error for specific initial conditions and set-points.

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